Calculus I: Understanding the Concept of Rate of Change

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Objectives:

- Use an applet of a moving object to collect data of a moving object.
- Learn about position function using real data.
- Explain the relationship between the position, velocity and acceleration functions of a moving object.
- Relate the concept of derivative in real life situations as a rate of change.

Introduction:

When an object is released from rest, has zero velocity. Right after release, the object picks up a velocity as it moves downward. The velocity of the object moving downward is assigned a negative sign. The magnitude (absolute value) of the velocity which is called *speed* increases with time. For a given period of time, the average velocity is defined to be the ratio of the distance travelled during the given time period to the time interval. Velocity is the rate of change of position per unit of time. Speed is the magnitude of velocity. Acceleration is the rate of change of velocity per unit of time.

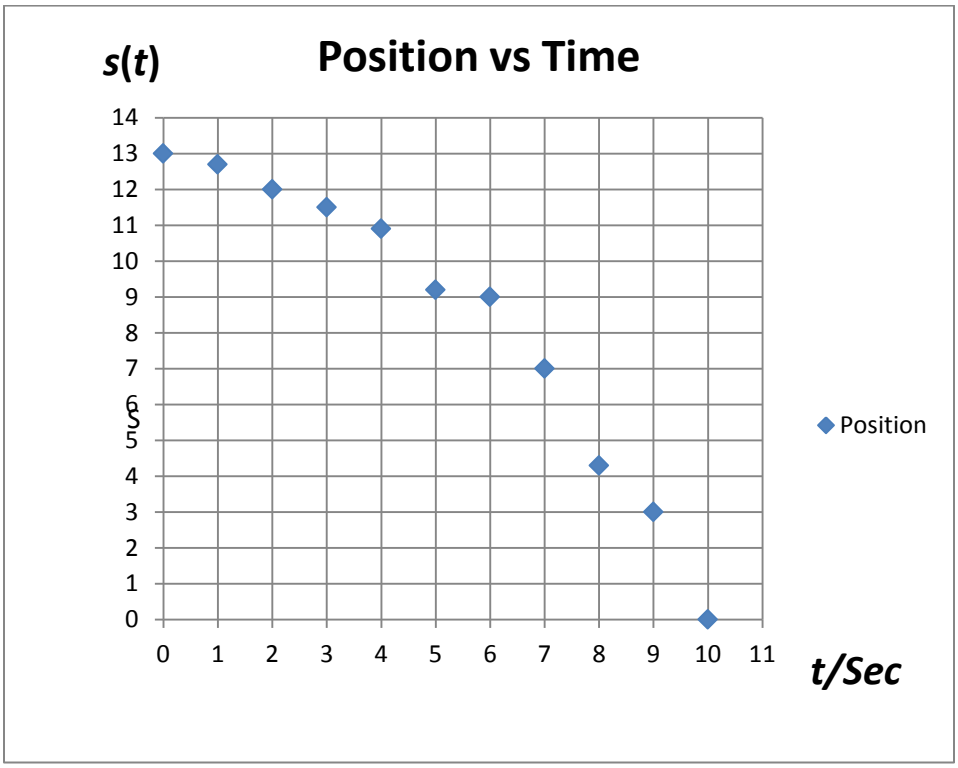
Lab Activity:

A free fall applet will be used to record position vs. time data of a falling object. MS Excel, Mathematica or MATLAB programs will be used for graphing and modeling.

a) Use an applets to model a moving object
b) Collect data of the position and plot the data using Excel or MATLAB
c) Find equation of the best fit curves for both position and velocity as a functions of time
d) Estimate the average velocity and acceleration using the data
e) Calculate velocity and acceleration using formulas
f) Compare the results

Free Fall Activity

The following scatter diagram depicts data that were obtained from a falling object and shows the position, $s(t)$ in meters vs. time, t, in seconds.



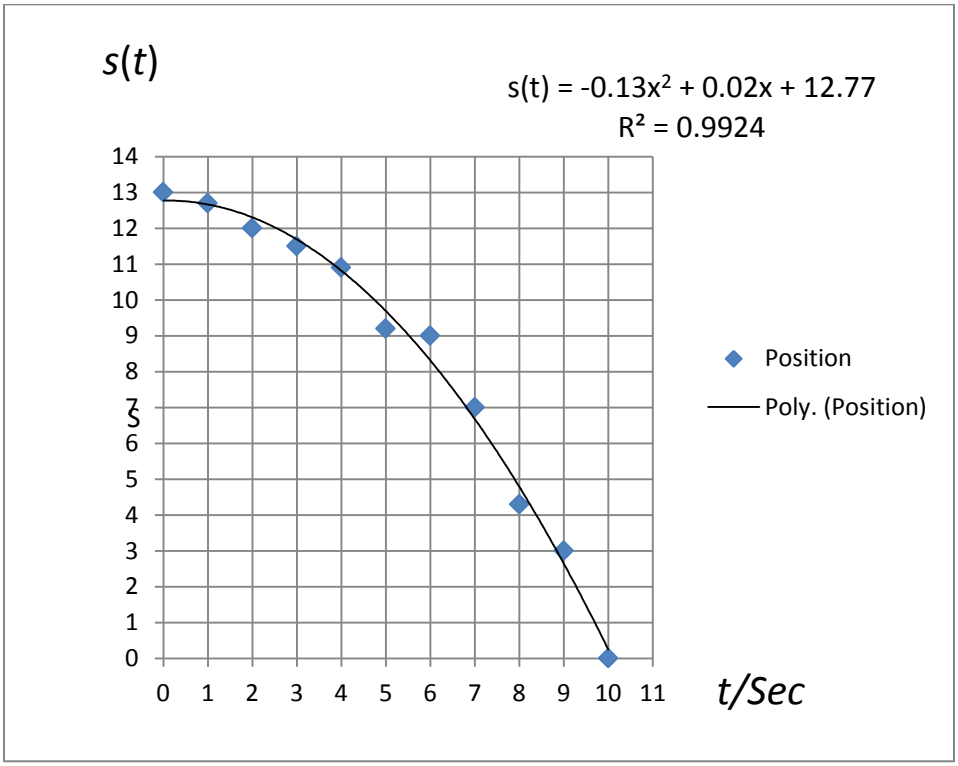
(1) Visually, what kind of model (i.e., linear, quadratic, cubic, logarithmic, exponential…) would be an appropriate model for the data?

(2) The average velocity within the time interval of (t_1, t_2) is defined to be the slope of the second line passing through the 2 points $(t_1, s(t_1))and\ (t_2, s(t_2))$, i.e.:

$$V_{ave} = \frac{s(t_2) - s(t_1)}{t_2 - t_1} = \frac{\Delta s}{\Delta t}$$

Estimate the average velocity for the time interval of (4,7) from the above graph.

(3) Free fall follows a quadratic model. In the following graph, a parabola has been used to model the data. Use the equation of the position function to find the average velocity for the time interval (4,7). Compare your results with part (1), and comment.

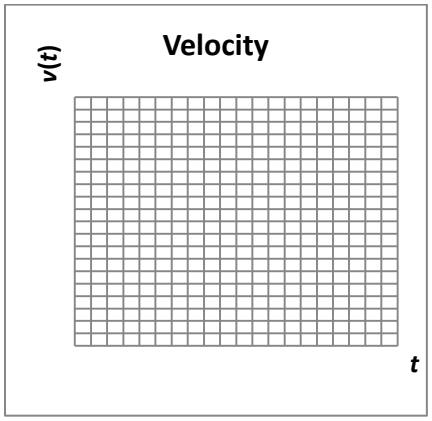


(4) Instantaneous velocity or velocity at a given point is the defined to be the slope of the tangent line to the graph of position function at the given time.
Estimate the velocity at $t = 4$ using the graph.

(5) As the second line approaches the tangent line at the pt. *a*, the value of the average velocity approaches the velocity (slope of tangent line) at the pt. *a*. Therefore, the velocity at *a*, is the derivative of the position at *a*.

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = s'(t)$$

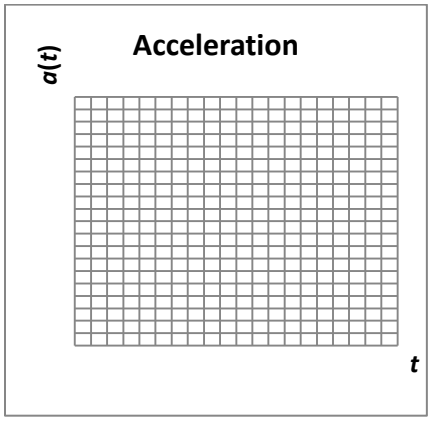
Find a general formula for the velocity function and graph it.



(6) Using the equation of the velocity function calculate the velocity at t=4. Compare the result with part (4) and comment.

(7) Explain why the velocity is a straight line with a negative slope. Interpret the meaning of velocity as it relates to the position of a moving object.

(8) Acceleration, $a(t)$ is the rate of change of velocity which is the slope of tangent line to the velocity function at each pt. Find the acceleration function, $a(t)$ and graph it. What is $a(4)$ and what does it represent?



(9) Is the acceleration constant or changing? Interpret the meaning of acceleration as it relates to the velocity of a moving object.